Waves in Ultrasonics

Author: David R Andrews

Cambridge Ultrasonics

28th January 2011

Ultrasonic waves are mechanical waves either in the bulk of a material or on the surface.

All equations predicting the speed of sound in bulk materials have a general form of

$$Speed = \sqrt{\frac{stiffness}{density}}$$

This is a consequence of the wave equation for ultrasonic and acoustic waves. In one-dimension the general form of the wave equation is

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{E}\frac{\partial^2 u}{\partial t^2}$$

General solutions (d'Alembert's solution) predict two waves travelling at speed c in both the positive x-direction and the negative x-direction.

$$u = u(x \pm ct)$$

When either solution is used then the following relationship emerges

$$1 = \frac{\rho}{E}c^2 \qquad \text{or} \qquad c = \sqrt{\frac{E}{\rho}}$$

Note the stiffness/density as mentioned above.

Explicit solutions have been found for many systems with rectangular, spherical and cylindrical geometry. Where an explicit solution cannot be found then computational methods such as the finite element and the finite difference methods can be used. They represent the vibrating system by point masses and springs located on a mesh spanning the volume of the system. Computers can be used to calculate approximate solutions.

A. Reflection, transmission, refraction and mode conversion

When a wave in one material passes through an interface into a second material some energy is reflected and some is transmitted. By considering the continuity of the amplitudes of waves normal and parallel to the interface it can be shown that the reflected intensity is a proportion R of the incident intensity $R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$

Where Z_1 is the acoustic impedance of the first material and Z_2 is the acoustic impedance of the second material and acoustic impedance equals the product of density and wave speed. The proportion of the beam transmitted, T, is given by

$$T = 1 - R$$

If the angle of incidence is oblique and the angle with the normal is θ then the reflected beam is at an angle ϕ to the normal and

$$\theta = \phi$$

Snell's law applies to the refraction of ultrasonic and acoustic waves, with θ_2 the angle of refraction to the interface, c_1 the speed of sound in the material supporting the incident wave and c_2 the speed of sound in the material supporting the refracted wave.

$$\frac{\sin\theta}{\sin\theta_2} = \frac{c_1}{c_2}$$

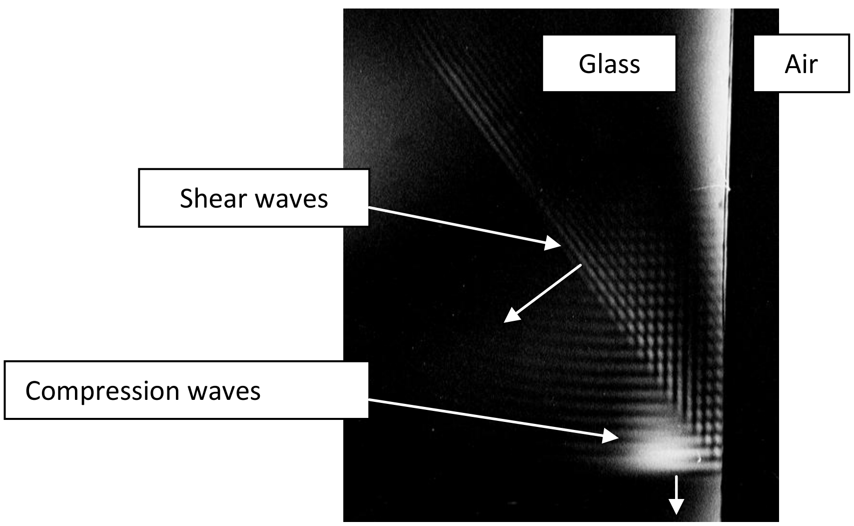


Figure 1

A photograph of ultrasonic waves rendered visible in glass. Compression waves at 2.5 MHz are travelling downwards, grazing a free surface where mode-conversion creates shear waves at the same frequency. Since the speed of the shear waves is lower, the wavelength is shorter and they are emitted at an angle to the compression waves.

B. Transducer beams

When a wave is launched from a transducer it is interesting to know how it travels thereafter. Firstly, plane waves emerge, with the same width as the transducer and, secondly, waves are created at its perimeter, known as edge waves. If the transducer is circular then the edge wave is an expanding toroid, this wave can be seen in cross-section in figures 2 and 3.

Close to the transducer, within a distance L, in the *near field*, the edge waves interfere with each other and with the plane waves, resulting in rapidly varying amplitudes with distance. Further away, in the *far field*, the edge waves are always approximately tangential to the plane waves in the centre of the beam and only constructive interference can occur.

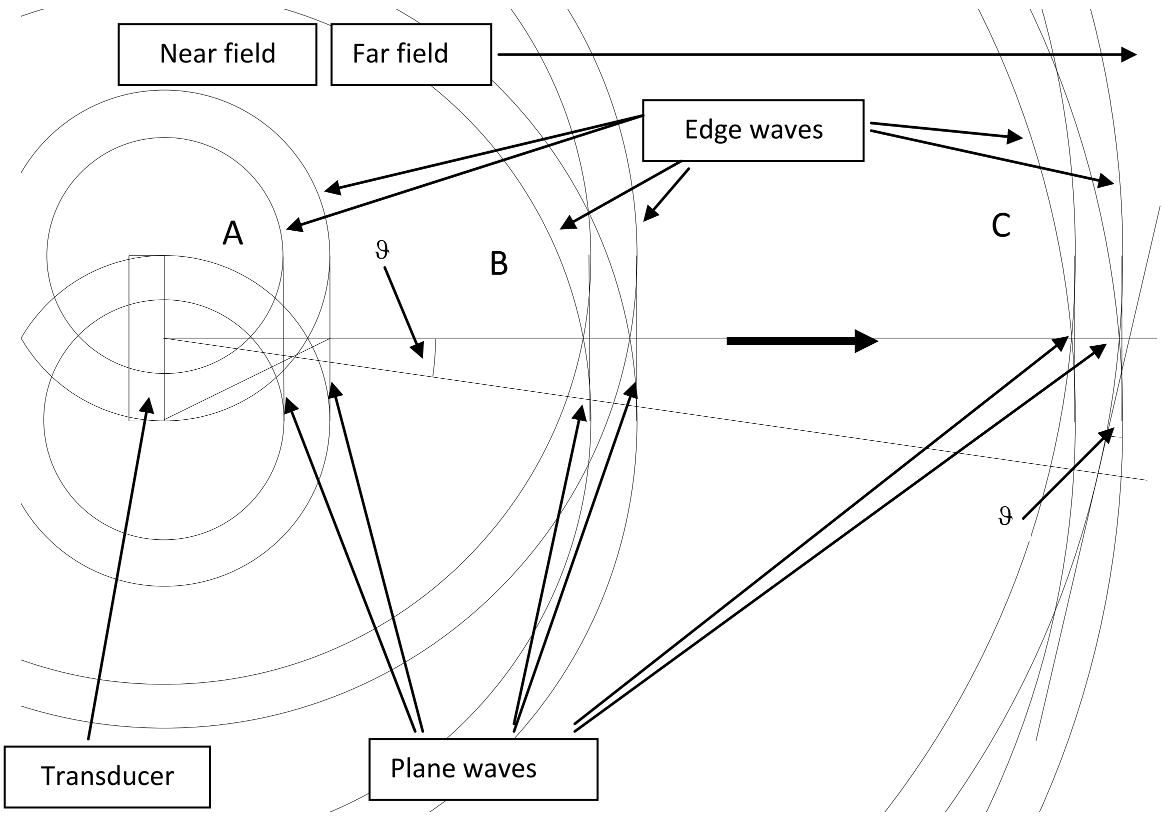


Figure 2

Sketch illustrating how waves emerge (travelling from left to right) from a transducer and cross three regions: A near field, B and C the far field.

Interference in the centre of the far field is always constructive if the edge waves are within half the wavelength $\lambda/2$ of the plane wave. This criterion can be used to predict the range of the near field, L, using simple geometry. If the aperture of the transducer is D then

$$\lambda/2 < L - \sqrt{L^2 - \left(\frac{D}{2}\right)^2}$$

Or to a first approximation

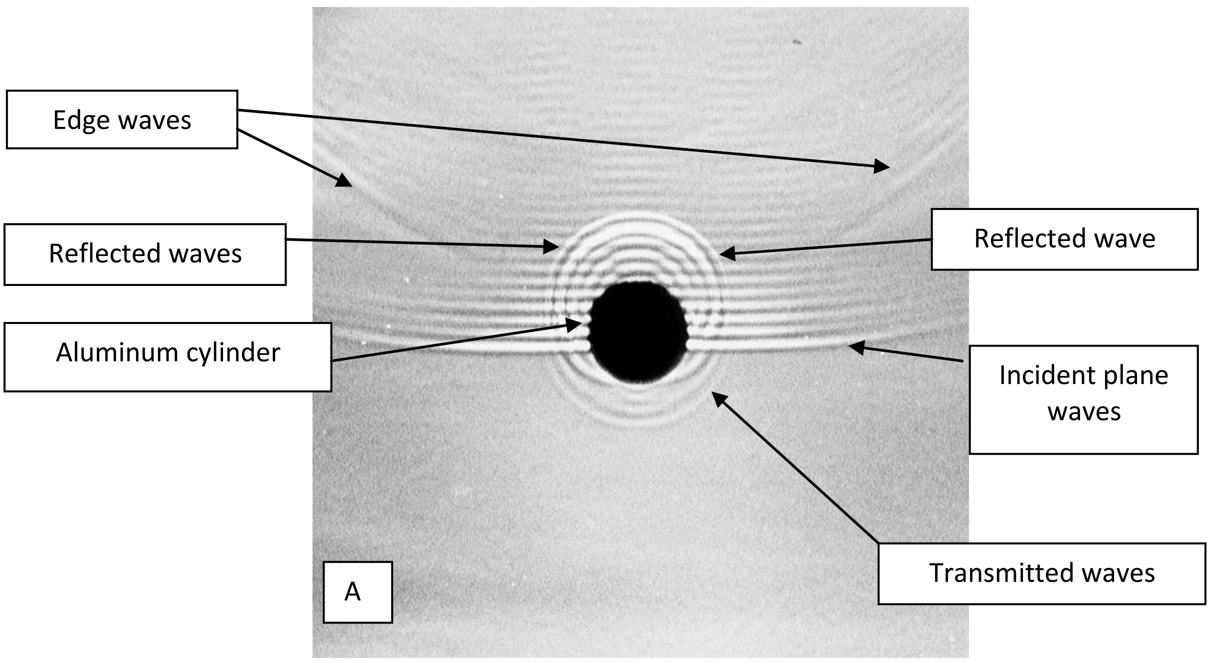
$$L < \frac{D^2}{4\lambda}$$

At the side of the beam in the far field, the edge waves from opposite sides of the aperture interfere destructively creating an amplitude null, or node, in the shape of a cone of semi-angle ϑ. Outside this cone the interference is successively constructive and destructive. Simple geometry can be used to estimate ϑ.

$$\sin\vartheta \approx {}^{\lambda/2}\!/_{D/2} = \frac{\lambda}{D}$$

More rigorous arguments show that

$$\sin\vartheta = 1.22\frac{\lambda}{D}$$



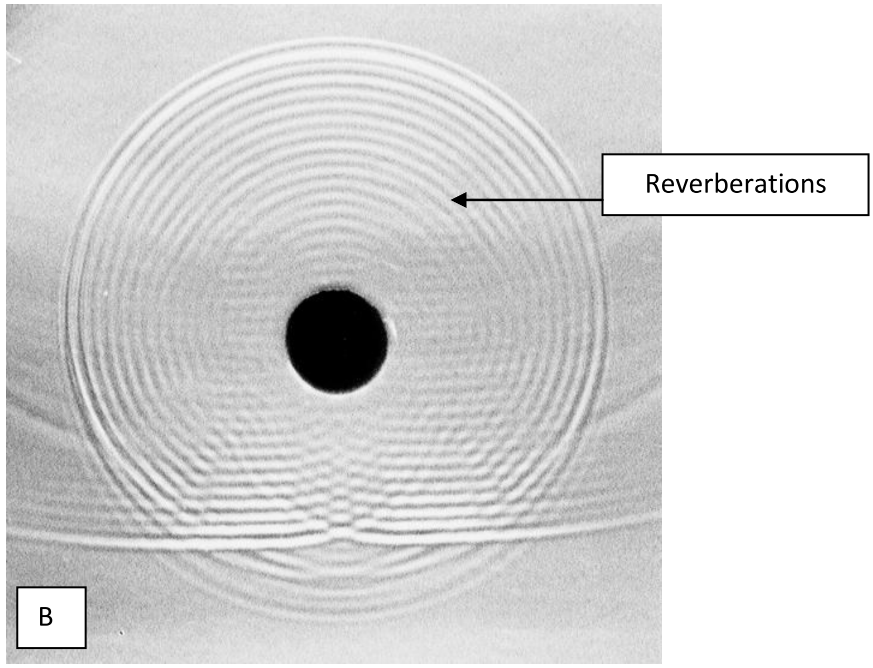


Figure 3

Two photographs showing ultrasonic plane waves and edge waves (1.5 MHz) rendered visible in water. Photograph B shows waves approximately 10 μs after the first. The solid, black circle is an aluminium cylinder viewed along its length. The photographs show the effects of reflection, transmission and reverberation.

C. Attenuation

A mechanical wave can lose energy by two principal mechanisms: thermoelastic losses in homogeneous materials (energy is converted into heat) or scattering in heterogeneous materials (the wave is scattered in many directions). Both mechanisms result in an gradual loss of intensity as the wave travels - the effect is referred to as attenuation. Scattering does not convert ultrasonic wave energy into another form of energy so no energy is lost by this mechanism but the wavefront loses coherence. The value of attenuation is proportional to frequency squared for thermoelastic losses. The way attenuation is measured can strongly influence the value when scattering is the dominant mechanism. The randomly distributed aggregate particles in concrete, for example, are random scatterers if the wavelength is equal to or less than the size of the aggregates. If attenuation is measured using a pair of 50 mm diameter transducers at 200 kHz (wavelength is approximately 20 mm) then the receiver will register a small signal, indicating high attenuation. The same experiment performed on a homogeneous material, like aluminum, results in a much larger signal. Concrete has a higher attenuation than aluminum, measured this way. However, when an array of 10 mm diameter receivers is used as an energy detector then a different, much lower value of attenuation is measured for concrete (see figure 4). In medical diagnostic compound B-mode images random scattering is referred to as speckle.

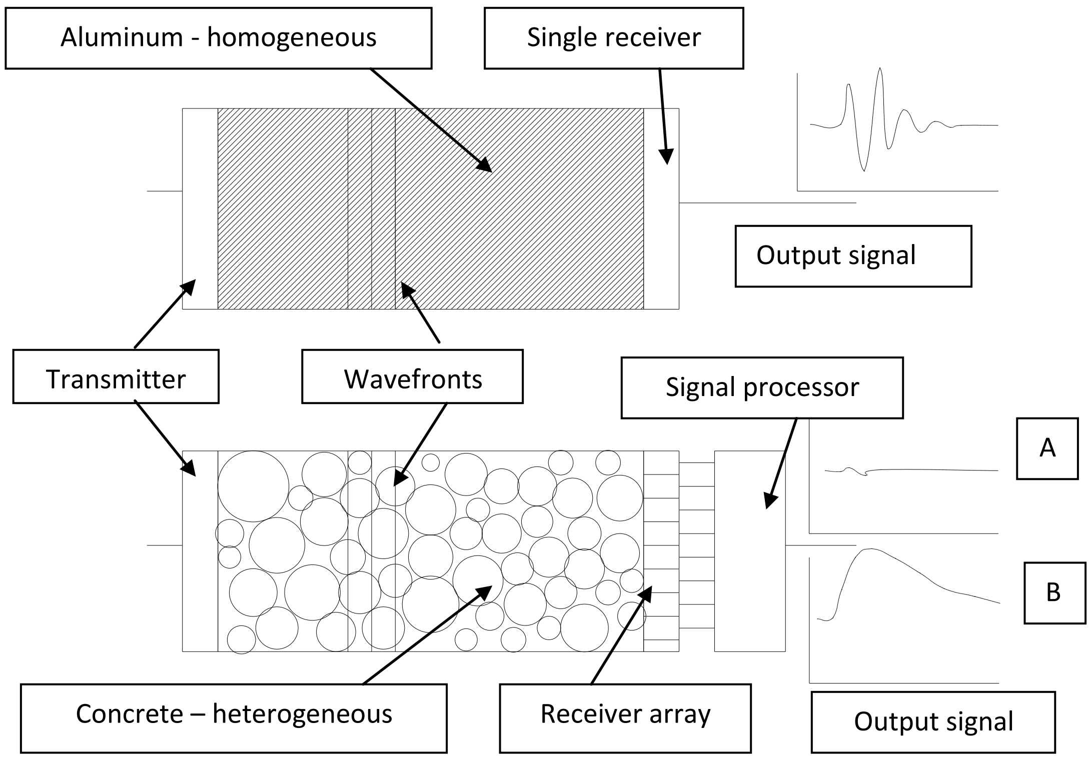


Figure 4

Sketch to illustrate the effect of scattering losses in an attenuation measurement and the effect of receiver type on the result. The homogeneous material gives a strong signal with a single, coherent receiver. A heterogeneous material gives a low output (A) with a coherent receiver, indicating high attenuation. However, the heterogenous material gives a strong signal (B) with an energy detector, indicating low attenuation. Signal processor in A – simple summing circuit. Signal processor in B – envelope detector followed by summation.

Attenuation, α, is formally measured in nepers m^{-1} but it is more commonly measured in dB m^{-1} or sometimes in dB m^{-1} Hz^{-1}. The neper is a consequence of the exponential decay of intensity with distance.

α (db m^{-1}) = $20(\log_{10}e)$ (neper m^{-1})

It is also common to compare materials in terms of attenuation per wavelength (measured in dB) because the size of an experiment or ultrasonic system is usually an important underlying factor to consider and, generally, the size of the experiment in proportion to the wavelength. Table IV gives some typical attenuation values.

	Attenuation α (db m^{-1})	**Attenuation /wavelength (db)**	**Attenuation α (neper m^{-1})**
Water 1 MHz	0.22	1.5×10^{2}	2.5×10^{-2}
Water 1 GHz	2.2×10^{5}	1.5×10^{11}	2.5×10^{4}
Blood 1 MHz	18	1.2×10^{4}	2.1
Air 1 MHz	1200	3.6×10^{6}	138
Aluminium 1 GHz	7500	1.3×10^{9}	860

Table I

Variation of attenuation for four materials.

D. Doppler effect

When ultrasound of frequency F is reflected from a scatterer, which is moving relative to the material supporting the ultrasonic wave, then the frequency of the reflected wave is changed. The amount the frequency is changed, F_d, is known as the Doppler-shift frequency and the value depends upon the vector velocity, $\underline{v}$, of the scatterer relative to the vector of the ultrasound frequency, $\underline{F}$, in the direction of travel of the ultrasonic wave (speed c). Where $\underline{v}$ is positive if it is measured in the same direction as $\underline{F}$.

$$F_d = -\frac{2}{c}\underline{v} \cdot \underline{F}$$

The Doppler effect is exploited in medical ultrasonic systems to measure the speed of flowing blood or the speed of heart valves opening and closing. It is also used in some flow meters for metering fluids, for example water or oil and gas. The Doppler shift frequency for a wave of 2 MHz passing through blood flowing at 10 cm s-1 at 60^{o} away from the transducer is –125 Hz (frequency reduction). The frequency of the returning echo will be 1999875 Hz. Colour is used in compound B-mode images to indicate the presence of Doppler shifts.

E. Dispersion

Experiments show that materials generally cause pulses of waves to become longer as they travel. The effect is called dispersion. Dispersion is caused by waves of different frequencies travelling at different speeds, with the lowest frequencies usually travelling fastest. Dispersion and attenuation are closely related. There are many causes of dispersion, including: the existence of boundaries to the material, particularly if they are regular or symmetrical; inherent material properties at the molecular and atomic level, associated with force transfer and effective mass; scattering of waves in heterogeneous materials; the dependence of wave speed on amplitude (non-linear dispersion).

A mathematical expression for a travelling wave is

$$\sin(kx - \omega t) + \sin(kx + \omega t)$$

Where x is the distance travelled, t is time, $k=2\pi/\lambda$ is the wave number (λ is the wavelength) and $\omega=2\pi F$ is the angular frequency (F is frequency). The phase velocity or the speed of a single frequency component in the wave is $v = F\lambda = \omega/k$. The speed at which energy or modulation moves along with the wave, v_g, is called the group velocity. It is given by

$$v_g = \frac{d\omega}{dk} = v + k\frac{dv}{dk}$$

The effect of dispersion on a pulse is progressive, depending upon the distance travelled in the material. Studying dispersion can be aided by using time-frequency representations of transmitted signals.

F. Sonar equation

The sonar equation is of fundamental importance in all ultrasonic and acoustic systems although it is more widely used in sonar design than in other applications. It is used to predict the voltage level, R_x, of an echo from a target given an electrical voltage drive, T_x, to the transmitter. In simplified form it is

$$R_x = T_x.C^2{}_x.\frac{S}{r^4}$$

Where Cx is the electromechanical conversion efficiency of the transducer, assumed here to be used both for transmission and reception hence raised to power of two, S is the strength of the scatterer (how much incident energy it reflects back) and r is the range to the scatterer, assuming an omni-directional transducer in a deep ocean. The power of the transmitter is spread out over a spherically expanding shell of area $4\pi r^2$ as it travels to the scatterer. At the scatterer some power is reflected back and it becomes a secondary source, creating a second spherically expanding shell of area $4\pi r^2$, which travels back to the transmitter/receiver. It is the combined effect of the two expanding shells that accounts for the $1/r^4$ term. Factors can also be included to allow for attenuation during transmission.

J. Derived SI units encountered in ultrasonics and acoustics

Table V lists the majority of units used in the field of ultrasonics and acoustics.

Impedance = density x speed

Intensity = energy /area

Quantity	Unit name	Unit symbol	Derivation
Attenuation			neper m^{-1} (or dB m^{-1})
Density			Kg m^{-3}
Frequency	Hertz	Hz	s^{-1}
Impedance	Rayl	Z	kg m^{-2} s^{-1}
Intensity			W m^{-2}
Pressure	Pascal	Pa	N m^{-2}
Speed			m s^{-1}
Wavelength			m

Table V

Quantities commonly used in the field of ultrasonics and acoustics.

BIBLIOGRAPHY

Auld, B.A., (1990) *Acoustic fields and waves*. Second Ed. Krieger Publishing Company, Florida.

Birks, A.S., Green, R.E., (eds) (1991) *Nondestructive testing handbook* Volume 7 Ultrasonic testing. 2nd Ed. Am Soc Nondestructive Testing.

Bushong, S.C., Archer, B.J., (1991) *Diagnostic ultrasound*. Mosby-Year Book, St Louis.

Coates, R.F.W., (1990) *Underwater acoustic systems*. MacMillan Education, Basingstoke.

Crampin, S., (1987). *Geological and industrial implications of extensive-dilatancy anisotropy*. Nature, 328

Fink, M., (1992). *Time reversal of ultrasonic fields – Part 1 Basic principles and 2 Experimental results*. IEE Trans Ultrason., Ferroelect., Freq., Contr 39, No 5

Kino, G.S., (1987) *Acoustic waves*. Prentice-Hall, New Jersey

Leighton, T.G., (1994) *The acoustic bubble*. Academic Press, San Diego

Lynn, P.A., (1987) *Radar systems* MacMillan Education, Basingstoke

Maynard, J,D., (1996) *Resonant ultrasound spectroscopy*. Physics Today 49