Fundamentals of Ultrasonics

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Mechanical vibrations and waves in solids, liquids and gases can be classed as ultrasonic or acoustic waves. Liquids and gases only support compression waves, represented by a scalar pressure, with a longitudinal particle motion in the same direction as the wave. In solids there can also be two orthogonal, transverse motions (vector shear strains), all of which can be combined into a single tensor representation. Ultrasonic and acoustic waves offer considerable scope for technological exploitation because of the diversity of wave modes, the ease with which waves can be launched and received, their ability to travel long distances and because the waves are inherently sensitive to mechanical structure. They have found application in many fields, most notably: medical diagnosis, non-destructive evaluation, geophysical exploration and sonar. Applications using pulsed waves are predominantly associated with inspection whereas applications using continuous waves are predominantly associated with material processing. The spectrum of acoustics and ultrasound covers three ranges of frequencies, the first from about 0.1 Hz to 20 Hz is sometimes termed *infrasound*; the second is the range of human hearing, generally taken to be 20 Hz to 20 kHz; the third extends beyond human hearing up to about 100 MHz – it is this third range that is commonly termed *ultrasound*. The term *acoustic* is not used with much precision, it is sometimes used to indicate the range of human hearing but acoustic is sometimes used to include infrasound and ultrasound as well. There are relatively few technological applications in the range of human hearing, apart from musical instruments and systems used to relay audible information to humans, the reason being, presumably, to avoid causing interference to human hearing. Experiments show that the attenuation of ultrasonic waves increases with increasing frequency for virtually all materials. At a frequency of 100 MHz the wavelength of ultrasound is so small, typically 50 μm in metals, 15 μm in liquids such as water and 3 μm in gases, that all materials have significantly high attenuation and 100 MHz is the approximate upper limit of technological applications of ultrasound.

I. Fundamentals.

Any system, comprising a collection of masses, that is able to apply forces between the masses can carry ultrasonic or acoustic waves. Distributed masses and point masses, quasi-static Hookean forces or very short duration collision forces are all examples of classes of systems capable of supporting acoustic waves. More specific examples of such systems are, at the microscopic scale, atoms and molecules in solids, liquids and gases; at a very large scale galaxies should be capable of supporting very low frequency waves.

A. One-dimensional system

The simplest system able to support ultrasonic or acoustic waves has only one dimension, such as a rope or string held taught at each end, where mass is continuously distributed along the length and the force is the line tension. A one-dimensional system can support the following two fundamental modes of vibration.

1. Longitudinal or compression waves (scalar)

The compression of the wave at any point along the string can be described by a scalar quantity. Particle motion is parallel to the direction of travel of the wave.

2. Transverse waves (vector)

The motion of particles in a transverse wave is perpendicular to the direction of travel of the wave. The transverse displacement is described by resolving it into two orthogonal planes. It is possible to have polarization states of transverse waves, in which two orthogonal waves of the same frequency and speed have a fixed phase relationship, for example: linear, circular and elliptical polarizations.

Real ropes and strings have a measurable thickness and can also support torsion vibrations, due to the moment of inertia and the shear modulus of the string. Compression (longitudinal scalar) and shear (transverse vector) are the two fundamental forces and motions in ultrasonic and acoustic waves.

B. More complex systems

Periodicity in a system causes periodicity in the vibration pattern and any solution must satisfy Floquet's principle.

$$F(z+d) = F(z)$$

Where F(z) describes the vibration pattern and d is the periodicity. A Fourier series of the following type is a solution because of Floquet's principle.

$$F(z) = \sum_{n=\propto}^{\propto} a_n e^{-i(2\pi n/d)z}$$

The jointed pipes used in a riser in an oil well are an example of a periodic structure. Floquet's principle shows the riser behaves like a filter to acoustic waves with periodic pass-bands and nulls (comb-filter).

In the one-dimensional systems considered so far the masses were distributed evenly and the forces obeyed Hooke's law. Force is proportional to extension in Hooke's law. Newton's law, relating force, mass and acceleration is used with Hooke's law to construct an equation of motion. The same principles are applied in three dimensions but tensor notation is used. The concept of a force is replaced by stress, the force per unit area, and the concept of extension is replaced by strain, the extension per unit length. Both stress and strain are tensor quantities and each contains components describing compression and shear.

C. Atomic models of wave transport mechanisms – speed of sound

Gas (at 273 K and 10^5 Pa)	Molecular weight	Speed of sound (ms^{-1})
Air	14	330
Carbon Dioxide	44	260
Deuterium	2	890
Hydrogen	1	1300
Hydrogen Bromide	81	200

Table I

Data for five gases of different molecular weights showing the variation of sound speed with molecular weight. Lighter molecules transport sound faster than heavier molecules.

Continuum models predict that the speed of sound, c, in a gas at moderate pressures is given by

$$c = \sqrt{\gamma RT / M}$$

Where γ is the ratio of specific heats at constant pressure and constant volume, T is the temperature in Kelvin and M is the molecular weight. This expression shows that the larger the mass of the molecule the more difficult it is to move it quickly and the lower will be the speed of sound. The factor γRT is a constant at to a fair approximation.

Liquid (at 293 K and 10^5 Pa)	Density (kg m^{-3})	Speed of sound (ms^{-1})
Ethyl alchol	789	1100
Helium (4.2 K)	120	183
Mercury	13590	1450
Sodium (383 K)	970	2500
Water	1000	1500

Table II

Data for five liquids of different densities showing the variation of sound speed with density.

It is known that the speed of sound, c, in a liquid is given by

$$c = \sqrt{1/\beta_a \rho}$$

Where β is the adiabatic compressibility and ρ is the density. Note the $1/\sqrt{\rho}$ dependency, which is the same kind of dependency as $1/\sqrt{M}$ for gases. In this case, β is not a constant, it is a material parameter with significant variability.

Solid (at 293 K and 10^5 Pa)	Density (kg m^{-3})	Speed of sound (longitudinal) (ms^{-1})	Speed of sound (transverse) (ms^{-1})	Speed of sound (surface) (ms^{-1})
Aluminum	2700	6400	3100	2900
Diamond	2300	18600		
Concrete	2400	2500 - 5000	1200 - 2500	1000 - 2000
Sapphire (Al_2O_3) z-axis	4000	11000	6000	
Steel – mild steel	7900	6000	3200	3000
Wood	650	3500		

Table III

Data for six solids showing the variation of sound speeds for three different wave modes: longitudinal, transverse and surface waves.

It is known that the longitudinal speed of sound, c_L, and the transverse speed of sound, c_T, in a solid are given by

$$c_L = \sqrt{E/\rho} \quad \text{and} \quad c_T = \sqrt{G/\rho}$$

Where E is Young's modulus, G is the shear modulus and ρ is the density. The last two expressions only apply to isotropic materials. It is always true that $c_L > c_T$. A surface wave travels along a surface of a material. The particle motion is elliptical with the greatest amplitude at the surface, decaying with depth. Surface waves travel at speeds approximately the same as shear waves.

BIBLIOGRAPHY

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